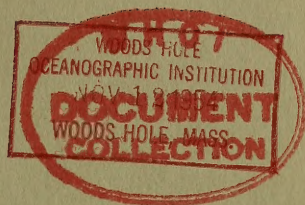


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DEPARTMENT OF ARMY
CORPS OF ENGINEERS
WASHINGTON, D. C.



A STATISTICAL STUDY OF THE EFFECT OF WAVE
STEEPNESS ON WAVE VELOCITY

by

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Theoretical investigations of the velocities of ocean waves have in general concluded that wave steepness (H/L) has a negligible effect on wave velocity when the wave steepness is small. Under these conditions, the wave velocity is given by

$$C_{H=0} = \left[-\frac{g}{k} \tanh kd \right]^{\frac{1}{2}} \quad (1)$$

where $C_{H=0}$ is the wave velocity neglecting wave steepness (Airy Theory) (ft/sec.)

g = Acceleration of gravity (ft/sec²)

$k = 2\pi/L$ (1/ft.)

d = Water depth (ft.)

This result can be derived from the AIRY theory(1)*, which neglects wave steepness, and from the theories of STOKES(1) and GERSTNER(1), which consider wave steepness.

When the wave steepness becomes large, Stokes theory as modified by STRUIK (2) and WOLF(3) predicts that the wave velocity will increase and gives the velocity to the second approximation as

$$C_T = C_{H=0} \cdot \left\{ \left[1 + \left(\frac{\pi H}{L} \right)^2 \right] \left[f \left(\frac{d}{L} \right) \right] \right\}^{\frac{1}{2}} \quad (2)$$

where C_T = the wave velocity considering wave steepness (Stokes theory) (ft/sec)

H = Wave height (ft)

L = Wave length (ft.)

and $f(d/L)$ is given by

$$f(d/L) = \frac{2 \cosh^2 \frac{4\pi d}{L} + 2 \cosh \frac{4\pi d}{L} + 5}{8 \sinh^4 \frac{2\pi d}{L}}$$

The present investigation seeks to determine the comparative degree of reliability of these theories in predicting the wave velocity.

* Numbers in parentheses refer to list at end of paper.

Table 1

Beach Erosion Board Data

| Run No. | d (ft.) | T (sec.) | L (ft.) | d/L | H (ft.) | H/L | C _H =0 ft/sec | C _T ft/sec | C _T ft/sec | C _m ft/sec |
|---------|---------|----------|---------|-------|---------|-------|--------------------------|-----------------------|-----------------------|-----------------------|
| 1 | 0.7 | 1.00 | 4.05 | 0.173 | 0.20 | 0.049 | 4.07 | 4.12 | 4.16 | 4.05 |
| 2 | 0.7 | 1.00 | 4.05 | 0.173 | 0.10 | 0.025 | 4.07 | 4.08 | 4.09 | 4.05 |
| 3 | 0.7 | 1.00 | 4.05 | 0.173 | 0.27 | 0.067 | 4.07 | 4.15 | 4.24 | 4.09 |
| 4 | 0.7 | 1.80 | 8.15 | 0.086 | 0.16 | 0.020 | 4.54 | 4.55 | 5.58 | 4.41 |
| 5 | 0.7 | 1.80 | 8.15 | 0.086 | 0.06 | 0.007 | 4.54 | 4.54 | 4.70 | 4.54 |
| 6 | 0.7 | 1.80 | 8.15 | 0.086 | 0.21 | 0.026 | 4.54 | 4.55 | 4.73 | 4.65 |
| 7 | 0.7 | 1.80 | 8.15 | 0.086 | 0.26 | 0.032 | 4.54 | 4.56 | 4.83 | 4.50 |
| 8 | 0.7 | 2.65 | 12.31 | 0.057 | 0.14 | 0.011 | 4.65 | 4.65 | 4.75 | 4.53 |
| 9 | 0.7 | 2.65 | 12.31 | 0.057 | 0.035 | 0.003 | 4.65 | 4.65 | 4.66 | 4.47 |
| 10 | 0.7 | 2.65 | 12.31 | 0.057 | 0.28 | 0.023 | 4.65 | 4.66 | 5.03 | 5.54 |
| 11 | 0.7 | 2.65 | 12.31 | 0.057 | 0.25 | 0.020 | 4.65 | 4.66 | 4.95 | 5.51 |
| 12 | 0.7 | 3.50 | 16.53 | 0.042 | 0.08 | 0.005 | 4.69 | 4.69 | 4.78 | 4.67 |
| 13 | 0.7 | 3.50 | 16.53 | 0.042 | 0.20 | 0.012 | 4.69 | 4.70 | 5.22 | 4.91 |
| 14 | 0.7 | 3.50 | 16.53 | 0.042 | 0.31 | 0.019 | 4.69 | 4.70 | 5.88 | 4.94 |
| 15 | 0.5 | 3.50 | 13.92 | 0.036 | 0.21 | 0.015 | 3.98 | 3.98 | 4.15 | 4.10 |
| 16 | 0.5 | 3.50 | 13.92 | 0.036 | 0.09 | 0.007 | 3.98 | 3.98 | 4.01 | 4.15 |
| 17 | 0.5 | 3.50 | 13.92 | 0.036 | 0.02 | 0.001 | 3.98 | 3.98 | 4.04 | 4.14 |
| 18 | 0.5 | 3.50 | 13.92 | 0.036 | 0.22 | 0.016 | 3.98 | 3.98 | 4.17 | 4.21 |
| 19 | 0.5 | 2.65 | 10.47 | 0.048 | 0.18 | 0.017 | 3.96 | 3.96 | 4.65 | 4.13 |
| 20 | 0.5 | 2.65 | 10.47 | 0.048 | 0.11 | 0.011 | 3.96 | 3.96 | 4.23 | 4.15 |
| 21 | 0.5 | 2.65 | 10.47 | 0.048 | 0.03 | 0.003 | 3.96 | 3.96 | 3.98 | 4.82 |
| 22 | 0.5 | 1.80 | 6.99 | 0.072 | 0.04 | 0.006 | 3.89 | 3.89 | 3.90 | 3.92 |
| 23 | 0.5 | 1.80 | 6.99 | 0.072 | 0.12 | 0.017 | 3.89 | 3.89 | 4.01 | 3.94 |
| 24 | 0.5 | 1.80 | 6.99 | 0.072 | 0.19 | 0.026 | 3.89 | 3.90 | 4.22 | 3.88 |
| 25 | 0.5 | 1.00 | 3.59 | 0.139 | 0.17 | 0.047 | 3.60 | 3.64 | 3.73 | 3.62 |
| 26 | 0.5 | 1.00 | 3.59 | 0.139 | 0.16 | 0.045 | 3.60 | 3.63 | 3.70 | 3.67 |
| 27 | 0.5 | 1.00 | 3.59 | 0.139 | 0.08 | 0.022 | 3.60 | 3.61 | 3.63 | 3.66 |
| 28 | 0.8 | 1.00 | 4.24 | 0.189 | 0.11 | 0.026 | 4.25 | 4.26 | 4.27 | 4.20 |
| 29 | 0.8 | 1.00 | 4.24 | 0.189 | 0.22 | 0.052 | 4.25 | 4.30 | 4.33 | 4.22 |
| 30 | 0.5 | 1.00 | 4.24 | 0.118 | 0.28 | 0.066 | 4.25 | 4.34 | 4.39 | 4.33 |
| 31 | 0.5 | 1.00 | 4.24 | 0.118 | 0.23 | 0.055 | 4.25 | 4.31 | 4.35 | 4.31 |
| 32 | 0.5 | 1.80 | 8.68 | 0.058 | 0.06 | 0.007 | 4.82 | 4.82 | 4.83 | 4.64 |
| 33 | 0.8 | 1.80 | 8.68 | 0.092 | 0.13 | 0.014 | 4.82 | 4.82 | 4.87 | 4.70 |
| 34 | 0.8 | 1.80 | 8.68 | 0.092 | 0.29 | 0.033 | 4.82 | 4.84 | 5.31 | 4.96 |
| 35 | 0.8 | 2.65 | 13.13 | 0.061 | 0.33 | 0.025 | 4.96 | 4.97 | 5.39 | 5.42 |
| 36 | 0.8 | 2.65 | 13.13 | 0.061 | 0.15 | 0.011 | 4.96 | 4.96 | 5.04 | 5.38 |
| 37 | 0.8 | 2.65 | 13.13 | 0.061 | 0.35 | 0.003 | 4.96 | 4.95 | 4.96 | 4.70 |
| 38 | 0.8 | 3.50 | 17.50 | 0.046 | 0.38 | 0.022 | 5.01 | 5.02 | 6.59 | 5.48 |
| 39 | 0.8 | 3.50 | 17.50 | 0.046 | 0.18 | 0.011 | 5.01 | 5.01 | 5.39 | 5.11 |
| 40 | 0.8 | 3.50 | 17.50 | 0.046 | 0.07 | 0.004 | 5.01 | 5.01 | 5.07 | 4.79 |
| 41 | 1.0 | 3.50 | 20.13 | 0.050 | 0.07 | 0.003 | 5.58 | 5.58 | 5.61 | 5.48 |
| 42 | 1.0 | 3.50 | 20.13 | 0.050 | 0.22 | 0.011 | 5.58 | 5.58 | 5.92 | 5.58 |
| 43 | 1.0 | 3.50 | 20.13 | 0.050 | 0.46 | 0.023 | 5.58 | 5.59 | 6.95 | 5.88 |
| 44 | 1.0 | 2.65 | 14.60 | 0.068 | 0.53 | 0.036 | 5.51 | 5.54 | 6.31 | 5.58 |
| 45 | 1.0 | 2.65 | 14.60 | 0.068 | 0.21 | 0.014 | 5.51 | 5.51 | 5.64 | 6.34 |
| 46 | 1.0 | 2.65 | 14.60 | 0.068 | 0.09 | 0.006 | 5.51 | 5.51 | 5.53 | 5.69 |
| 47 | 1.0 | 1.80 | 9.58 | 0.104 | 0.13 | 0.014 | 5.32 | 5.32 | 5.33 | 5.77 |
| 48 | 1.0 | 1.80 | 9.58 | 0.104 | 0.34 | 0.036 | 5.32 | 5.35 | 5.41 | 5.82 |
| 49 | 1.0 | 1.80 | 9.58 | 0.104 | 0.38 | 0.040 | 5.32 | 5.35 | 5.44 | 5.34 |
| 50 | 1.0 | 1.80 | 9.58 | 0.104 | 0.49 | 0.051 | 5.32 | 5.38 | 5.52 | 5.45 |
| 51 | 1.0 | 1.00 | 4.52 | 0.221 | 0.32 | 0.071 | 4.52 | 4.63 | 4.68 | 4.57 |
| 52 | 1.0 | 1.00 | 4.52 | 0.221 | 0.23 | 0.051 | 4.52 | 4.58 | 4.60 | 4.56 |
| 53 | 1.0 | 1.00 | 4.52 | 0.221 | 0.14 | 0.031 | 4.52 | 4.54 | 4.55 | 4.53 |
| 54 | 1.2 | 1.00 | 4.72 | 0.254 | 0.15 | 0.032 | 4.72 | 4.74 | 4.75 | 4.62 |
| 55 | 1.2 | 1.00 | 4.72 | 0.254 | 0.31 | 0.065 | 4.72 | 4.84 | 4.82 | 4.72 |
| 56 | 1.2 | 1.00 | 4.72 | 0.254 | 0.41 | 0.087 | 4.72 | 4.90 | 4.94 | 4.82 |
| 57 | 1.2 | 1.80 | 10.03 | 0.120 | 0.19 | 0.019 | 5.74 | 5.75 | 5.79 | 5.68 |
| 58 | 1.2 | 1.80 | 10.03 | 0.120 | 0.32 | 0.031 | 5.74 | 5.77 | 5.88 | 5.82 |
| 59 | 1.2 | 1.80 | 10.03 | 0.120 | 0.42 | 0.041 | 5.74 | 5.79 | 5.98 | 6.16 |
| 60 | 1.2 | 2.65 | 15.87 | 0.076 | 0.32 | 0.020 | 6.04 | 6.06 | 6.39 | 6.59 |
| 61 | 1.2 | 2.65 | 15.87 | 0.076 | 0.55 | 0.035 | 6.04 | 6.09 | 7.04 | 6.19 |
| 62 | 1.2 | 2.65 | 15.87 | 0.076 | 0.15 | 0.009 | 6.04 | 6.04 | 6.12 | 6.08 |
| 63 | 1.2 | 3.50 | 21.30 | 0.056 | 0.37 | 0.017 | 6.09 | 6.10 | 6.39 | 5.80 |
| 64 | 1.2 | 3.50 | 21.30 | 0.056 | 0.28 | 0.013 | 6.09 | 6.10 | 6.26 | 5.93 |
| 65 | 1.2 | 3.50 | 21.30 | 0.056 | 0.65 | 0.031 | 6.09 | 6.12 | 6.98 | 6.32 |

Table 2

University of California Data

| Run No. | d (ft.) | T (sec.) | L (ft.) | d/L | H (ft.) | H/L | C _{H=0} ft/sec | C _T ft/sec | C _T ft/sec | C _m ft/sec |
|---------|---------|----------|---------|--------|---------|-------|-------------------------|-----------------------|-----------------------|-----------------------|
| 2 | 2.00 | 0.514 | 1.49 | 1.342 | 0.076 | 0.051 | 2.76 | 2.80 | 2.80 | 2.89 |
| 4 | 2.00 | 0.507 | 1.49 | 1.342 | 0.112 | 0.095 | 2.76 | 2.90 | 2.90 | 2.94 |
| 3 | 2.00 | 0.517 | 1.53 | 1.307 | 0.126 | 0.081 | 2.80 | 2.89 | 2.89 | 2.95 |
| 7 | 2.00 | 0.597 | 1.98 | 1.010 | 0.118 | 0.060 | 3.19 | 3.24 | 3.25 | 3.32 |
| 8 | 2.00 | 0.597 | 2.04 | 0.980 | 0.167 | 0.082 | 3.23 | 3.34 | 3.34 | 3.41 |
| 9 | 2.00 | 0.597 | 2.10 | 0.952 | 0.205 | 0.098 | 3.28 | 3.43 | 3.43 | 3.52 |
| 12 | 2.00 | 0.571 | 2.53 | 0.790 | 0.162 | 0.064 | 3.60 | 3.68 | 3.68 | 3.77 |
| 176 | 1.00 | 0.573 | 1.28 | 0.781 | 0.115 | 0.090 | 2.56 | 2.65 | 2.66 | 2.70 |
| 14 | 2.00 | 0.693 | 2.61 | 0.766 | 0.243 | 0.093 | 3.65 | 3.81 | 3.81 | 3.77 |
| 13 | 2.00 | 0.677 | 2.77 | 0.722 | 0.207 | 0.075 | 3.76 | 3.86 | 3.88 | 4.09 |
| 15 | 2.00 | 0.682 | 2.91 | 0.687 | 0.297 | 0.102 | 3.86 | 4.06 | 4.06 | 4.27 |
| 225 | 1.99 | 0.867 | 3.81 | 0.524 | 0.362 | 0.095 | 4.11 | 4.50 | 4.50 | 4.39 |
| 186 | 1.00 | 0.593 | 1.99 | 0.502 | 0.208 | 0.105 | 3.19 | 3.36 | 3.36 | 3.37 |
| 127 | 2.03 | 0.858 | 4.06 | 0.500 | 0.436 | 0.108 | 4.54 | 4.80 | 4.80 | 4.74 |
| 20 | 2.00 | 0.863 | 4.01 | 0.499 | 0.276 | 0.069 | 4.51 | 4.62 | 4.63 | 4.64 |
| 21 | 2.00 | 0.867 | 4.01 | 0.499 | 0.324 | 0.081 | 4.31 | 4.43 | 4.43 | 4.62 |
| 19 | 2.00 | 0.852 | 4.08 | 0.490 | 0.231 | 0.057 | 4.57 | 4.64 | 4.64 | 4.79 |
| 22 | 2.00 | 0.867 | 4.13 | 0.484 | 0.381 | 0.092 | 4.59 | 4.78 | 4.78 | 4.76 |
| 30 | 2.00 | 0.970 | 4.38 | 0.457 | 0.464 | 0.106 | 4.72 | 4.97 | 4.99 | 4.52 |
| 23 | 2.00 | 0.863 | 4.11 | 0.454 | 0.412 | 0.093 | 4.73 | 4.93 | 4.94 | 5.11 |
| 128 | 2.03 | 0.916 | 4.59 | 0.443 | 0.477 | 0.104 | 4.83 | 5.08 | 5.10 | 5.00 |
| 219 | 0.91 | 0.678 | 2.19 | 0.413 | 0.037 | 0.017 | 3.23 | 3.28 | 3.28 | 3.23 |
| 27 | 2.00 | 0.958 | 5.01 | 0.399 | 0.271 | 0.054 | 5.03 | 5.10 | 5.12 | 5.23 |
| 187 | 1.00 | 0.734 | 2.64 | 0.379 | 0.224 | 0.085 | 3.64 | 3.77 | 3.78 | 3.60 |
| 69 | 1.01 | 0.717 | 2.73 | 0.370 | 0.256 | 0.094 | 3.70 | 3.86 | 3.96 | 3.82 |
| 28 | 2.00 | 0.958 | 5.46 | 0.366 | 0.330 | 0.060 | 5.23 | 5.32 | 5.34 | 5.70 |
| 29 | 2.00 | 0.967 | 5.51 | 0.363 | 0.405 | 0.074 | 5.25 | 5.39 | 5.41 | 5.70 |
| 129 | 2.03 | 1.050 | 5.72 | 0.355 | 0.448 | 0.078 | 5.34 | 5.50 | 5.55 | 5.45 |
| 39 | 2.00 | 1.135 | 5.70 | 0.351 | 0.496 | 0.087 | 5.33 | 5.53 | 5.55 | 5.03 |
| 36 | 2.00 | 1.137 | 6.20 | 0.323 | 0.372 | 0.060 | 5.54 | 5.64 | 5.65 | 5.45 |
| 35 | 2.00 | 1.140 | 6.34 | 0.315 | 0.318 | 0.050 | 5.59 | 5.66 | 5.68 | 5.56 |
| 37 | 2.00 | 1.133 | 6.84 | 0.292 | 0.423 | 0.062 | 5.77 | 5.88 | 5.90 | 6.04 |
| 38 | 2.00 | 1.133 | 7.13 | 0.281 | 0.468 | 0.066 | 5.87 | 5.99 | 6.01 | 6.29 |
| 131 | 2.03 | 1.300 | 8.45 | 0.273 | 0.455 | 0.061 | 6.37 | 6.10 | 6.12 | 5.74 |
| 171 | 1.98 | 1.288 | 7.40 | 0.268 | 0.434 | 0.059 | 5.95 | 6.05 | 6.08 | 5.74 |
| 122 | 1.99 | 1.300 | 7.50 | 0.265 | 0.360 | 0.048 | 5.98 | 6.05 | 6.07 | 5.77 |
| 211 | 2.00 | 1.420 | 7.99 | 0.250 | 0.353 | 0.044 | 6.12 | 6.18 | 6.20 | 5.63 |
| 212 | 2.00 | 1.437 | 8.09 | 0.247 | 0.393 | 0.049 | 6.15 | 6.22 | 6.25 | 5.63 |
| 64 | 1.01 | 0.966 | 4.30 | 0.235 | 0.342 | 0.080 | 4.45 | 4.60 | 4.64 | 4.45 |
| 67 | 1.01 | 0.966 | 4.33 | 0.233 | 0.349 | 0.081 | 4.47 | 4.61 | 4.66 | 4.48 |
| 66 | 1.01 | 0.966 | 4.35 | 0.232 | 0.342 | 0.079 | 4.47 | 4.61 | 4.66 | 4.50 |
| 65 | 1.01 | 0.975 | 4.38 | 0.231 | 0.349 | 0.080 | 4.47 | 4.62 | 4.67 | 4.50 |
| 213 | 2.00 | 1.437 | 8.74 | 0.229 | 0.465 | 0.053 | 6.32 | 6.40 | 6.44 | 6.08 |
| 132 | 2.03 | 1.340 | 9.10 | 0.223 | 0.436 | 0.048 | 6.43 | 6.50 | 6.53 | 6.90 |
| 172 | 1.96 | 1.375 | 9.01 | 0.218 | 0.363 | 0.040 | 6.37 | 6.42 | 6.45 | 6.55 |
| 75 | 1.01 | 1.000 | 4.68 | 0.216 | 0.242 | 0.058 | 4.58 | 4.66 | 4.70 | 4.68 |
| 76 | 1.01 | 1.030 | 4.74 | 0.213 | 0.240 | 0.051 | 4.60 | 4.65 | 4.69 | 4.61 |
| 124 | 1.99 | 1.460 | 9.38 | 0.212 | 0.346 | 0.037 | 6.46 | 6.50 | 6.54 | 6.43 |
| 189 | 1.00 | 1.040 | 4.74 | 0.212 | 0.240 | 0.051 | 4.60 | 4.66 | 4.69 | 4.56 |
| 55 | 1.016 | 1.180 | 5.62 | 0.181 | 0.363 | 0.065 | 4.84 | 4.94 | 5.03 | 4.76 |
| 190 | 1.00 | 1.220 | 5.56 | 0.180 | 0.228 | 0.041 | 4.85 | 4.88 | 4.89 | 4.56 |
| 135 | 2.03 | 1.720 | 11.30 | 0.179 | 0.475 | 0.042 | 6.84 | 6.90 | 6.97 | 6.55 |
| 51 | 1.016 | 1.160 | 5.72 | 0.178 | 0.355 | 0.062 | 4.90 | 4.99 | 5.04 | 4.94 |
| 82 | 1.016 | 1.200 | 6.05 | 0.168 | 0.233 | 0.039 | 4.93 | 4.95 | 4.99 | 5.04 |
| 191 | 1.00 | 1.380 | 6.45 | 0.155 | 0.237 | 0.037 | 4.98 | 5.01 | 5.08 | 4.68 |
| 217 | 0.58 | 0.930 | 3.86 | 0.151 | 0.317 | 0.082 | 3.82 | 3.94 | 4.20 | 4.15 |
| 56 | 1.01 | 1.350 | 6.80 | 0.149 | 0.378 | 0.056 | 5.06 | 5.13 | 5.27 | 5.04 |
| 58 | 1.01 | 1.330 | 6.87 | 0.147 | 0.378 | 0.055 | 5.06 | 5.14 | 5.29 | 5.16 |
| 59 | 1.01 | 1.350 | 6.89 | 0.147 | 0.377 | 0.055 | 5.07 | 5.14 | 5.28 | 5.11 |
| 220 | 0.91 | 1.182 | 6.17 | 0.147 | 0.088 | 0.014 | 4.80 | 4.81 | 4.94 | 5.22 |
| 109 | 0.78 | 1.180 | 5.52 | 0.141 | 0.310 | 0.056 | 4.48 | 4.55 | 4.71 | 4.66 |
| 110 | 0.78 | 1.215 | 5.60 | 0.139 | 0.315 | 0.056 | 4.49 | 4.56 | 4.73 | 4.61 |
| 224 | 0.49 | 0.983 | 3.76 | 0.131 | 0.038 | 0.010 | 3.61 | 3.62 | 3.68 | 3.83 |
| 107 | 0.88 | 1.380 | 6.90 | 0.127 | 0.385 | 0.056 | 4.84 | 4.91 | 5.14 | 5.00 |
| 61 | 1.01 | 1.600 | 8.42 | 0.120 | 0.390 | 0.046 | 5.25 | 5.30 | 5.52 | 5.26 |
| 60 | 1.01 | 1.550 | 8.60 | 0.117 | 0.395 | 0.046 | 5.25 | 5.30 | 5.57 | 5.54 |
| 221 | 0.91 | 1.733 | 9.45 | 0.0959 | 0.093 | 0.010 | 5.11 | 5.11 | 5.35 | 5.45 |

The experimental data presented in this study were taken from two sources. The data presented in Table 1 were taken in the steel wave tank of the Beach Erosion Board. This tank is 96 feet long, $1\frac{1}{2}$ feet wide and 2 feet deep. The other source of data is a report by MORISON(4) who made similar measurements in a tank 60 feet long, 1 foot wide and 3 feet deep at the University of California Fluid Mechanics Laboratory. The data shown in Table 2 were taken from Morison's report with the exception of the values for $C_{H=0}$ and C_T which were computed from the wave characteristics as given by Morison. It should be noted that the wave lengths used in the computation of the theoretical velocities for the Beach Erosion Board data were computed from the measured water depth and wave period, (T) using the Airy theory, while the wave lengths used in the computation of the theoretical velocities for the University of California data were computed from the measured wave velocity (C_m) and the measured wave period.

The measured velocities (C_m) shown in Table 1 were obtained by placing two parallel-wire gages(5) 10 feet apart in the wave tank and recording the wave profile from these gages on a dual-channel Brush recorder. A timing device made four marks per second on the record of the wave profiles. Thus, knowing the distance between the gages and the time required for the wave to travel from one gage to the next, the average wave velocity between the gages was determined.

Since there was considerable variation between the measured and theoretical velocities, a statistical comparison was made which consisted of finding the correlation coefficient(6) for each set of theoretical values and the measured values. The correlation coefficients were found for the Beach Erosion Board data independently, the Morison data independently, and for the combination of the two. The results of this comparison are shown in Table 3.

TABLE 3

Correlation Coefficients

| <u>Correlation Between</u> | <u>For BEB Data</u> | <u>For Morison Data</u> | <u>For Combination</u> |
|--------------------------------|-------------------------|-----------------------------|----------------------------|
| C_m vs $C_H = 0$ | .95 | .97 | .96 |
| C_m vs C_T | .91 | .98 | .95 |
| C_m vs C_T' | .95 | .97 | .96 |

From the correlation coefficients shown, it can be seen that theory neglecting wave steepness (eq. 1) predicts the wave velocities very well, since a correlation coefficient of 1.00 represents perfect correlation. When effect of wave steepness is included in the theory (eq. 2), the correlation coefficient for the Beach Erosion Board data decreases and the correlation coefficient for the University of California data increases, indicating poorer and better, respectively,

agreement of the measured data with theory. Since the theoretical velocity correction for the effect of wave steepness contains two functions, $f(d/L)$ and $f(H/L)$, another comparison was made to determine the relative effect of these two functions. This was done by computing the velocity with only $f(H/L)$ included from

$$C_T' = C_{H=0} \cdot \left[1 + \left(\frac{\pi H}{L} \right)^2 \right] \quad (3)$$

The correlation coefficients for C_T' and C_m were computed and are shown in Table 3. From these correlation coefficients and the graphs shown in Figures 1A, 1B, 3A and 3B, it can be seen that the $f(H/L)$ parameter does not have enough effect on the theoretical wave velocity to change the statistical correlation coefficient in relation to the correlation coefficient between $C_{H=0}$ and C_m . Therefore, the $f(d/L)$ parameter has the predominant effect in making the correlation coefficient poorer for the Beach Erosion Board data and better for the University of California data, using the correlations between C_m and $C_H = 0$ as a reference.

The effect of $f(d/L)$ on the theoretical wave velocity is more readily apparent from a plot of $f(d/L)$ versus (d/L) MORISON shown in Figure 4. If the University of California data are considered, the range of d/L is 0.0959 to 1.342. Over most of this range, $f(d/L)$ is 1.0, but varies from 9.8 to 1.0 over a comparatively small part of the range. This has the effect of making the theoretical velocities slightly larger as a whole (Figure 2B), and making the correlation between C_T and C_m higher than the correlation between $C_{H=0}$ and C_m . The range of d/L for the Beach Erosion Board data is from 0.036 to 0.254 and $f(d/L)$ varies from 398 to 1.3, which introduces too large a correction for wave steepness and the wave velocities are over corrected (Figure 2A). The over correction of the wave velocities is also revealed in the lowered correlation coefficient between C_T and C_m when compared to the correlation between $C_{H=0}$ and C_m .

It is interesting to note that in both Figures 1A and 1B, which compare C_m and $C_{H=0}$, there is a decided tendency for the predicted velocities to be too small, indicating that some correction toward larger predicted velocities is needed.

The conclusions which can be drawn from this study are:

1. The Airy theory predicts wave velocities well enough for most purposes, but is inadequate if accuracy in the order of 1% is desired.
2. The Stokes Theory, neglecting $f(d/L)$, gives negligible improvement over the Airy Theory.
3. The Stokes Theory, including $f(d/L)$, significantly improves upon the Airy Theory for $0.1 \leq d/L \leq 0.4$ and overcorrects the wave velocity for $d/L \leq 0.1$.

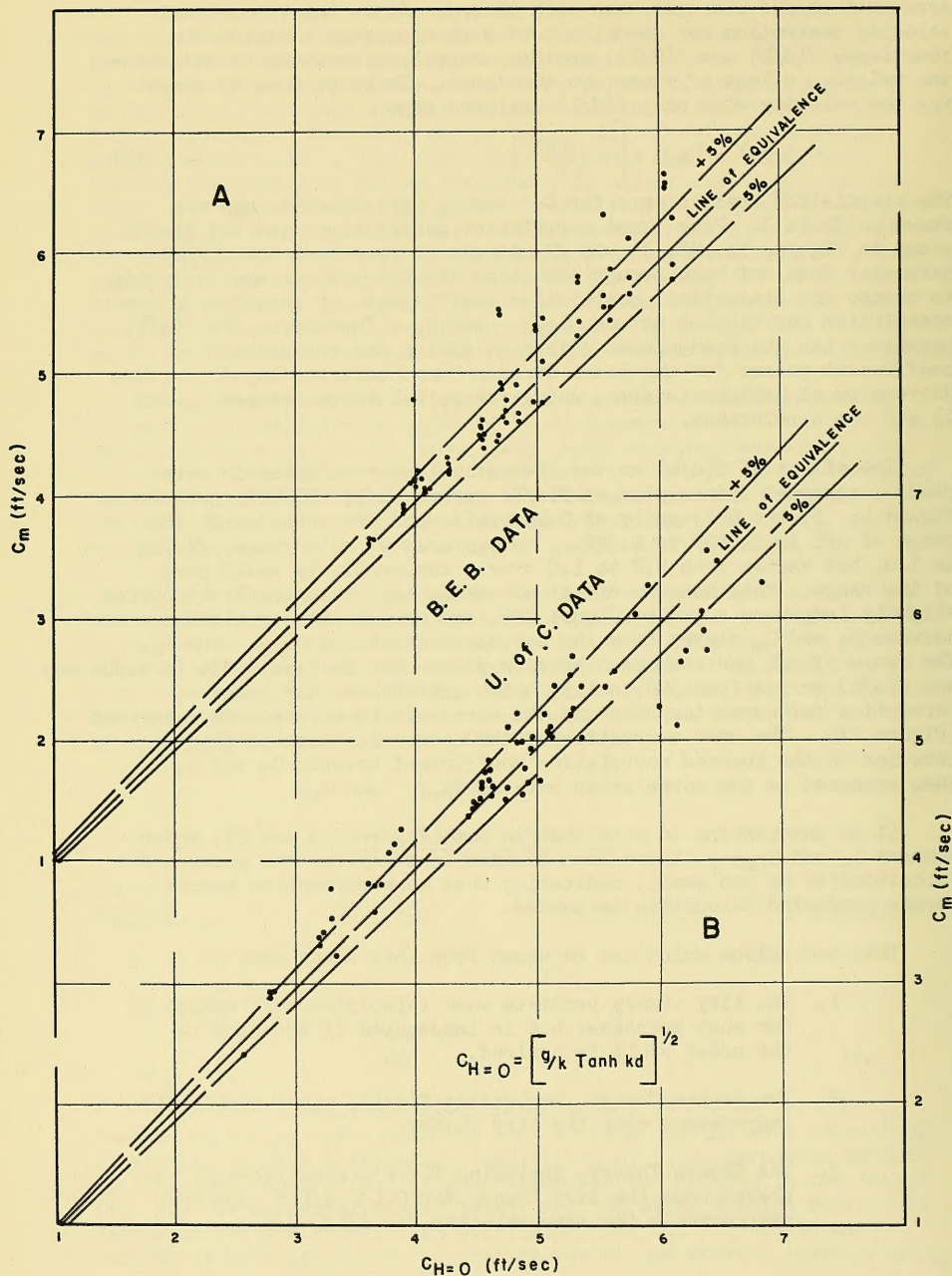


FIGURE 1. C_m VERSUS $C_{H=O}$

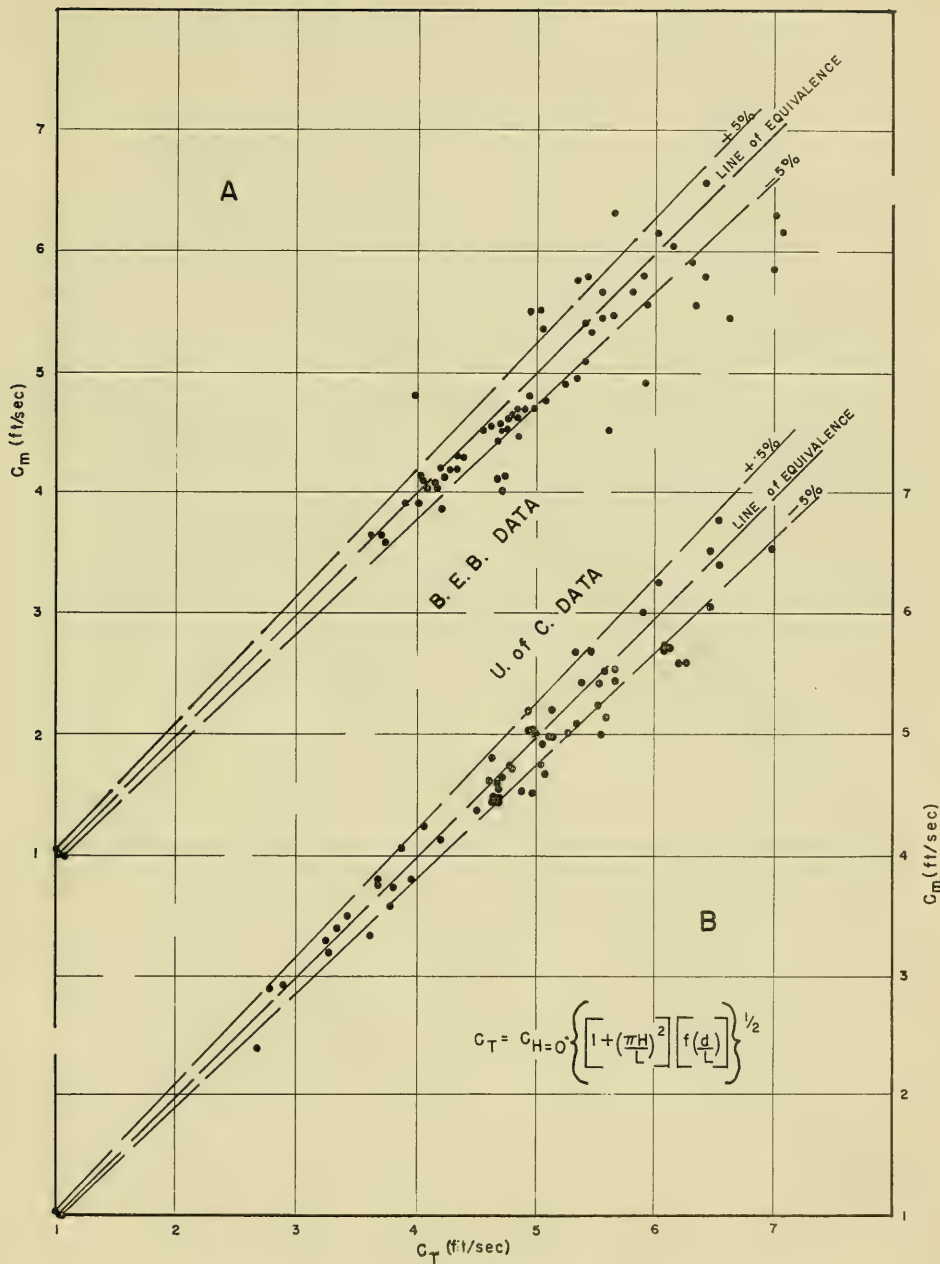


FIGURE 2. C_m VERSUS C_T

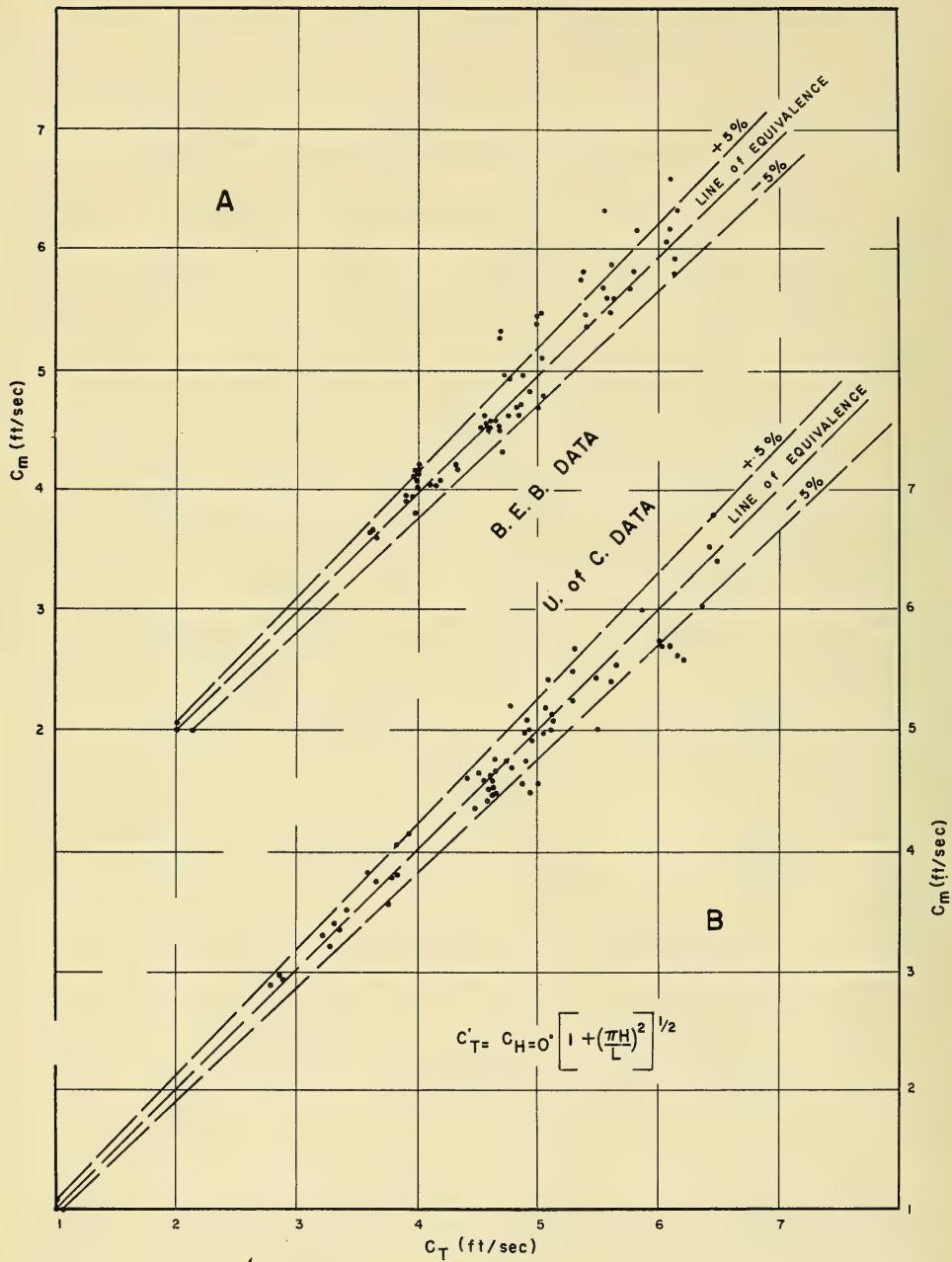


FIGURE 3. C_m VERSUS C'_T

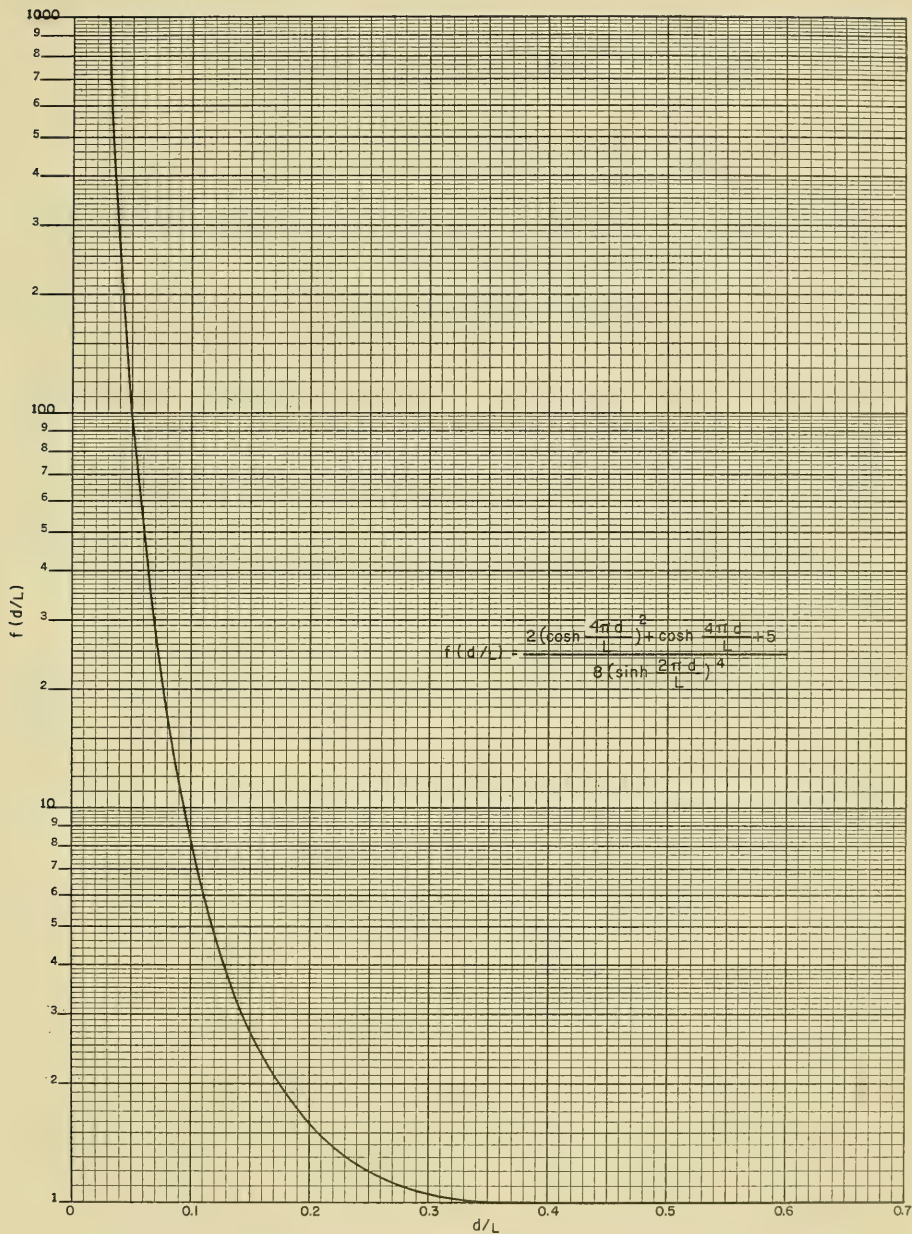


FIGURE 4. $f(d/L)$ VERSUS d/L

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TRAVELLING FORELANDS AND THE SHORE LINE
PROCESSES ASSOCIATED WITH THEM

by

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The shore formation here under consideration is that classified by Johnson(1) as a truncated cusped foreland. It is a V-shaped area of lowland that projects from a shore and travels along it consistently in one direction. A generalized sketch of a travelling foreland is shown in Figure 1. The front and back of the foreland are both gently curved, the front making a steeper angle with the shore than the back. The foreland may terminate in either a sharp or a blunt point. Back of the front and parallel thereto there lies a series of beach ridges, each of which represents a former front of the foreland.

The foreland travels because wave action is continually removing beach material from the back and depositing it in the form of successive beach ridges on the front. To understand how wave action produces these results it is necessary to consider the part played by the angle of incidence of the waves in determining their drift-producing power.

In Figure 2 there is shown a wave crest ab approaching a shore line cd with an angle of incidence i . On inspection it is seen that unless $i = 0$ the length of shore cd upon which the wave breaks is longer than the crest length ab . Hence the energy expended on each foot of shore line must be correspondingly less than the available energy per foot of wave crest. In fact, it is reasonable to suppose that, for a wave of given size, the energy expended per foot of shore line varies with the cosine of the angle of incidence, since that function represents the ratio of the lengths ab and cd .

Now, for the wave to move beach material alongshore it should have an alongshore component. Thus, if the angle of incidence were zero, there would be no shore drift although the expenditure of wave energy per foot of shore line would in that case be a maximum. Since the alongshore component of a wave of given size varies with the sine of the angle of incidence and since the energy it expends upon the shore varies with the cosine of the same angle, it follows that the drift-producing power of the wave varies with the product of these two functions, i.e., with $\sin i \cos i$. But $\sin i \cos i$ reaches its maximum value when $i = 45^\circ$. Hence waves of a given size will produce the heaviest shore drift when they strike the shore with an angle of incidence of 45° . These results should, of course, be accepted as approximate only.

(1) Johnson, D. W. - Shore Processes and Shore Line Development, John Wiley and Sons, 1919.

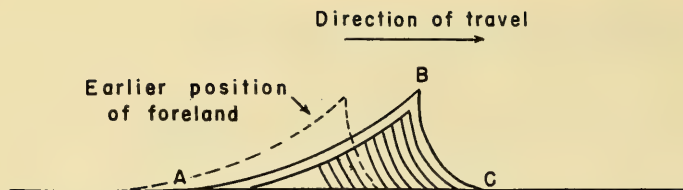


FIGURE 1. TRAVELLING FORELAND

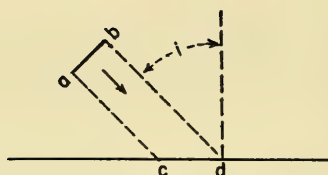


FIGURE 2. WAVE APPROACHING SHORE

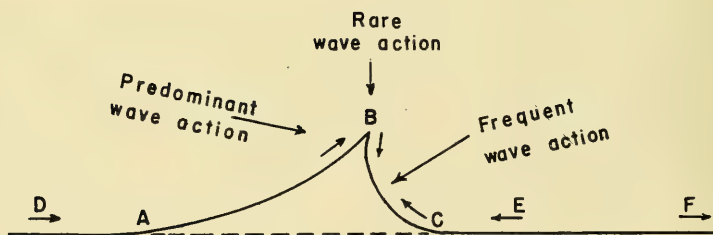


FIGURE 3. WAVE ACTION ON TRAVELLING FORELAND

The concept of a most favorable drift-producing angle is of great value in explaining a number of shore line phenomena. Its application in the explanation of the travelling foreland is illustrated in Figure 3. It is assumed that the predominant wave action is from the left, that wave action comes frequently from the right, but that waves from directly offshore are rare. The curvature of the back AB is such that the angle of incidence of the predominant wave action may approximate 45° at B. Hence the quantity of beach material in motion increases progressively from A to B. This increase can only take place at the expense of the beach, and therefore the back AB is eroded. On the front BC the angle of incidence of the waves from the right is normal near the center and is so oriented elsewhere as to produce a movement of material toward the center of the front. Hence accretion takes place on this face of the foreland.

It should be noticed that although the predominant wave action from the left produces a drift toward the right along most of the main shore, as is illustrated at points D and F of Figure 3, an opposite drift is produced at E which is in the lee of the foreland and hence protected from this wave action. As a result, beach material moves toward the foreland from both sides. The constant accumulation of beach material on the front of the foreland exceeds that eroded from the back, and the foreland increases in size as it advances.

Observation shows that travelling forelands ordinarily occur on the shores of long, narrow bodies of water. This is what might be expected as such a body would provide an adequate expanse of water for the generation by the wind of waves running with its longer axis but not at right angles to it. The direction of travel of the foreland would depend on the relative strength and frequency of the winds that blow approximately parallel to the long axis and upon the relative expanse of water to either side of the foreland. Figure 4 shows Cove Point, Calvert County, Maryland, an example of a travelling foreland on the west shore of Chesapeake Bay.



FIGURE 4. AN EXAMPLE OF A TRAVELLING FORELAND
COVE POINT, MARYLAND

PROGRESS REPORTS ON RESEARCH SPONSORED BY
THE BEACH EROSION BOARD

Abstracts from progress reports on several research contracts in force between universities or other institutions and the Beach Erosion Board, together with brief statements as to the status of research projects being prosecuted in the laboratory of the Beach Erosion Board are presented as follows:

I. University of California, Contract No. DA-49-055-eng-8, Status Report No. 15, 1 May 1954 through 1 July 1954

The statistical compilations of the data from sand samples collected around the rocky promontories in Southern California are now 75 percent completed. A similar statistical compilation of the analyses of sand samples collected along the Point Reyes beach is 60 percent completed. The mechanical analysis of the samples collected during the expedition at Santa Barbara in April is now completed and the data are in the process of compilation.

A report by Theodore Scott entitled "Sand Movement by Waves", was completed and published as Technical Memorandum No. 48 of the Beach Erosion Board. This report presents the results of a flume study of the effect of long and short period waves upon the formation of bars immediately adjacent to the coast line. In connection with this report, a great many significant observations upon the mechanics of sand transport, both on and offshore, as the results of wave action, were obtained.

II. University of California, Contract No. DA-49-055-eng-31, Status Report No. 4, 1 May 1954 to 31 July 1954

The analysis of the data for set-up is complete. The results indicate: (a) A rapidly increasing set-up with decreasing still-water depth; (b) There were no definite indications that the bottom roughness affects the set-up for relatively deep water; (c) In very shallow water the set-up is higher, the rougher the bottom. The trend is especially definite for higher wind velocities. For the shallowest still-water depth (0.05 foot) used in these experiments, the set-up was approximately 10 percent higher for the rough bottom and approximately 20 percent higher when strips of cheese cloth were used in the channel to simulate the roughness effects of vegetation.

Observations indicate, however, that the time necessary to reach the maximum set-up is considerably longer for the rough bottom conditions, and it may be possible that the maximums are usually not reached in nature where the duration of very high wind velocities is usually relatively very short.

The wave data have been evaluated and analyzed for the smooth bottom conditions, and partly completed for the rough bottom and the case with "vegetation".

The data indicate that Sverdrup-Munk-Bretschneider curves may be used to predict the wave heights for relatively deep water. As is to be expected in shallow water, however, the wave heights are considerably lower than predicted by the curves, depending upon the depth of the water. The experimental results indicate that the depth starts to affect the wave heights at approximately $d/H_0 < 5$. Here d is the actual depth at the location of wave measurements and H_0 the deep-water wave height as predicted by the use of above named curves for the given fetch and wind velocity.

The wave period in the generating area is also affected by the depth of water resulting in shorter wave periods than predicted by Sverdrup-Munk curves. The experimental results indicate that the depth starts to affect the wave period at approximately $d/L_0 < 0.2$. Here d is again the actual depth of the location of wave measurements and L_0 the deep-water wave length as computed by the formula $L_0 = 5.12 T_0^2$, where T_0 is the wave period for deep water and could be predicted by the curves.

Further it was found that the maximum wave height for a group of 100 waves is 1.34 times that of significant wave height $H_{1/3}$ for the same group. This is exactly the same value that was found for the Lake Okeechobee wave data and is lower than found for deep water in oceans (approximately 1.60).

The relationship between the mean and significant wave heights was found to be $H_{\text{mean}} = 0.68H_{1/3}$, which is somewhat higher than the value of 0.60 for Lake Okeechobee waves.

III. Scripps Institution of Oceanography, Contract No. DA-49-055-eng-3, Quarterly Progress Report No. 20, April to June 1954

Frequency-distribution curves have been computed for the maximum onshore and offshore components of orbital velocity, and for the heights and periods of the waves generating them.

Periodic measurements of sand-level changes with reference rods have now extended over a period of 15 months at three of the stations. The maximum changes during this period were 0.12, 0.15, and 0.06 foot in areas where the water depth is approximately 30, 52, and 70 feet respectively.

During the past three months the valley heads leading into Scripps Canyon have continued to shoal with a fill amounting to a maximum of 4 feet bringing these valleys back close to the highest points on record. However, there was some localized deepening in Summer Valley which amounted to $4\frac{1}{2}$ feet. This deepening did not persist in the adjacent lines but the record seems to leave little doubt of its having occurred.

The continued fill in New Valley supports the contention that the area may have become stabilized so that this valley may be completely filled returning to the conditions in 1949 when no valley existed in that vicinity.

IV. Massachusetts Institute of Technology, Contract No. DA-49-055-eng-16,

Additional data were gathered on the mean (sand) particle velocity along the beach and its relation to the wave and depth characteristics; and on the location of the null point (point separating the areas of onshore and offshore movement) and its relation to wave characteristics. The data tend to indicate a sudden reduction in drag force at one point which may be explained by a laminar-turbulent boundary layer transition on the sphere -- the break occurring at Reynold's numbers of the same order as those determined by Li.

V. Waterways Experiment Station, Vicksburg, Mississippi

Wave Run-up Study: Tests with the 1 on 3 smooth slope were completed.

Effect of Inlets on Adjacent Beaches: About 140 tidal cycles have been run under the new test set-up (the same as the previous set up, except for a shallow lagoon rather than a deep lagoon). The reaction appears to be very nearly identical to the first test, although resulting in a slightly wider and shallower channel, except for the time involved - things happening only about one-half as fast as in the first test.

VI. Beach Erosion Board, Research Division, Project Status Report for Quarter ending 15 September 1954.

In addition to the research projects under contract to various institutions which are reported on above, the Research Division of the Beach Erosion Board is carrying out certain projects with its own facilities. The main unclassified projects have been described in previous numbers of the Bulletin, and a short description of some of the work accomplished through the last quarter is given below.

Study of Effect of Tsunamis: The study was completed, and is to be published as a Technical Memorandum, showing the relation between wave height at the shore line and wave height in deeper water as a function of steepness, and between these variables and wave run-up on idealized shore structures. Actually the ratio of run-up to wave height at the shore line appears to approximate a constant value.

Groin Study: Difficulties in adequately trapping and measuring the sand transport (estimated to be on the order of 800-900 pounds per hour) without disturbing the wave pattern adjacent to the trapping area appear to be essentially overcome, and actual testing should start in the next quarter.

Routine progress, testing and analysis have been made on the other projects being carried out by the Research Division. In addition, reports on "Stability of Laminar Flow Along a Wall" by Huon Li, "Sand Movement by Waves" by T. Scott, and "Bore Hole Studies of the Naturally Impounded Fill at Santa Barbara, California", by P. D. Trask were received on contract reports and published as Technical Memorandums No. 47, 48 and 49 respectively.

BEACH EROSION STUDIES

Beach erosion control studies of specific localities are usually made by the Corps of Engineers in cooperation with appropriate agencies of the various States by authority of Section 2 of the River and Harbor Act approved 3 July 1930. By executive ruling the costs of these studies are divided equally between the United States and the cooperating agencies. Information concerning the initiation of a cooperative study may be obtained from any District or Division Engineer of the Corps of Engineers. A list of authorized cooperative studies follows:

AUTHORIZED COOPERATIVE BEACH EROSION STUDIES

MASSACHUSETTS

PEMBERTON POINT TO GURNET POINT. Cooperating Agency: Department of Public Works.

Problem: To determine the most suitable methods of shore protection, prevention of further erosion and improvement of beaches, and specifically to develop plans for protection of Crescent Beach, the Glades, North Scituate Beach and Brant Rock.

CONNECTICUT

STATE OF CONNECTICUT: Cooperating Agency: State of Connecticut (Acting through the Flood Control and Water Policy Commission)

Problem: To determine the most suitable methods of stabilizing and improving the shore line. Sections of the coast are being studied in order of priority as requested by the cooperating agency until the entire coast has been included.

NEW YORK

FIRE ISLAND INLET AND VICINITY: Cooperating Agency: Long Island State Park Commission

Problem: To determine the most practicable and economic method of providing adequate material to maintain the shore in a suitably stable condition and an adequate navigation channel at Fire Island Inlet.

N. Y. STATE PARKS ON LAKE ONTARIO. Cooperating Agency: Department of Conservation, Division of Parks.

Problem: To determine the best method of providing and maintaining certain beaches and preventing further erosion of the shore at the Braddock Bay area owned by the State of New York.

NEW JERSEY

STATE OF NEW JERSEY. Cooperating Agency: Department of Conservation and Economic Development.

Problem: To determine the best method of preventing further erosion and stabilizing and restoring the beaches, to recommend remedial measures, and to formulate a comprehensive plan for beach preservation or coastal protection. The current study covers the shore from Barnegat Inlet to Cape May.

DELAWARE

STATE OF DELAWARE. Cooperating Agency: State Highway Department.

Problem: To formulate a comprehensive plan for restoration of adequate protective and recreational beaches and a program for providing continued stability of the shores from Kits Hummock on Delaware Bay to Fenwick Island on the Atlantic Ocean.

NORTH CAROLINA

CAROLINA BEACH. Cooperating Agency: Town of Carolina Beach.

Problem: To determine the best method of preventing erosion of the beach.

CALIFORNIA

STATE OF CALIFORNIA. Cooperating Agency: Department of Public Works, Division of Water Resources, State of California

Problem: To conduct a study of the problems of beach erosion and shore protection along the entire coast of California. The current studies cover the Santa Cruz, Orange County and San Diego areas.

WISCONSIN

MANITOWOC-TWO RIVERS. Cooperating Agencies: Wisconsin State Highway Commission, Cities of Manitowoc and Two Rivers.

Problem: To determine the best method of shore protection and erosion control.

TERRITORY OF HAWAII

WAIMEA & HANAPEPE, KAUAI. Cooperating Agency: Board of Harbor Commissioners, Territory of Hawaii.

Problem: To determine the most suitable method of preventing erosion, and of increasing the usable recreational beach area, and to determine the extent of Federal aid in effecting the desired improvement.

